

Proposed Computer-Aided Design Algorithms for nth Order Chebyshev Active Filter.

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Abstract: This paper describes new algorithms for constructing the transfer function of the nth order Chebyshev active LPF by using the Cascade combination of second order and first order active filters. Using Computer-Aided with new algorithms for cascading s-domain transfer function of (conjugate poles and single real pole) can be achieved. Linear programming for performing the constructing of cascade combination is used by C++ program. The algorithms are very fast, flexible, and exact. Users of program can achieve the design of different components selection, cut-off frequency, magnitude of pass band ripple, and order of Chebyshev active LPF with very high flexibility.

Keywords: Programming algorithms, Active filter design, Frequency-dependent component.



1. INTRODUCTION

Analog filters are networks that process signals in a frequency-dependent manner. The basic concept of filter can be explained by examining the frequency dependent nature of the impedance of capacitors and inductors. That permits passing signals in certain frequencies and reject them in other frequencies.

An active filter is a type of analog filters that uses active components such as an amplifier. Active filter using operational amplifiers is an ideal circuit solution for many filter requirements. Active filters can be used for many applications.

Winder [1] has written a number of simple programs to help his work [1], his programs are like "Elsie, Passive (LC) filters only", "RC Filter" and "Filter Solutions".

Chen [2] has provided a lot of information in filters with significant additions in the areas of computer-aided design of active filters.

Pactitis[3] has introduced the theory and design of active filters. Raut [4] has also introduced the design of modern active filters and their analysis.

2. CHEBYSHEV LPF

Chebyshev filters are analog or digital filters having a steeper roll-off and more passband ripple than Butterworth filters. Therefore Chebyshev filters are used where the frequency content of a signal is more important than having a constant amplitude.

The normalized magnitude of prototype lowpass filter [1,2] is:

$$|H(jw/w_c)| = \frac{1}{\sqrt{1 + \epsilon^2 \cdot C_n^2(w/w_c)}} \dots\dots\dots(1)$$

Where C_n is nth order Chebyshev polynomial.
n is the order of the filter.

$$\epsilon = \sqrt{10^{-0.1A_p} - 1} \dots \dots \dots (2)$$

A_p is the magnitude of passband ripple.

The transfer function in s-domain can be given [3,4] as:

$$H(s) = \frac{H_0}{\prod_{k=1}^n (s - p_k)} \dots \dots \dots (3)$$

Where n is the order of the filter

p_k is the k th pole which can be calculated as shown:

$$p_k = -\sinh(\psi) \sin(\theta_k) + j \cosh(\psi) \cos(\theta_k) \dots \dots \dots (4)$$

Where

$$\theta_k = \frac{(2k-1)\pi}{2n} \dots \dots \dots (5)$$

$$\psi = \frac{1}{n} \sinh^{-1}\left(\frac{1}{\epsilon}\right) \dots \dots \dots (6)$$

Therefore from equation (4) it is clear that all poles are complex conjugate except for odd n there will be single real pole.

The multiplication of conjugate poles gives the following second order equation:

$$(s - p_k)(s - p_k^*) = s^2 + 2\text{real}(p_k)s + [\text{real}(p_k)]^2 + [\text{imag}(p_k)]^2 \dots \dots \dots (7)$$

Then

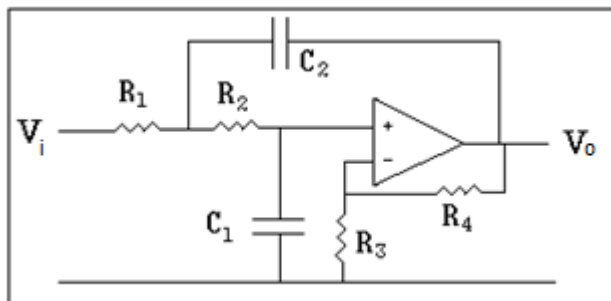
$$(s - p_k)(s - p_k^*) = s^2 + 2\left[\frac{\sinh(\psi) \sin(\theta_k)}{s + \left[\frac{\sinh(\psi) \sin(\theta_k)}{\cosh(\psi) \cos(\theta_k)} \right]^2} \right] \dots \dots \dots (8)$$

3. GENERAL CIRCUITS AND EQUATIONS

A low-pass filter is a device which permits the passing of signals of low frequencies and prevents the passing of those of high frequencies. There are many ways of designing LPFs using active devices.

A voltage-controlled voltage source (VCVS), which is shown in Figure (1), is a good way for second order LPF design which has the following transfer function [5,6,7,8]:

$$\frac{V_o(s)}{V_i(s)} = \frac{K}{s^2 + as + b} \dots \dots \dots (9)$$

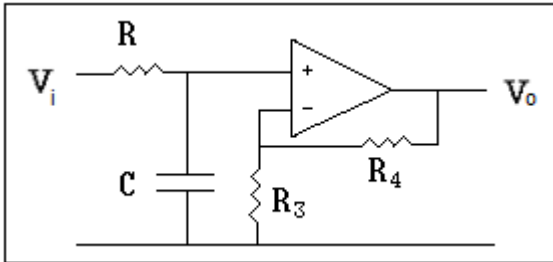


Figure(1) Second order active LPF

While the voltage –controlled voltage source (VCVS), which is shown in Figure (2), is a first order LPF which has the following transfer function[5,6,7,8]:

$$\frac{V_2(s)}{V_1(s)} = \frac{2B}{s + B} \dots\dots\dots(10)$$

(where R3=R4)



Figure(2) First order active LPF

An analysis of Fig(1) shows that it achieves Eq.(9) with:

$$K = \frac{2}{R_1 R_2 C_1 C_2} \dots\dots\dots(11)$$

(where R3=R4)

$$a = \frac{1}{R_1 C_2} \dots\dots\dots(12)$$

(where R3=R4)

$$b = \frac{1}{R_1 R_2 C_1 C_2} \dots\dots\dots(13)$$

An analysis of Fig.(2) shows that it achieves Eq.(10) with:

$$B = \frac{1}{RC} \dots\dots\dots(14)$$

Higher order filters may be obtained by cascading two or more second-order filters. To implement odd nth order, we can add a first order stage.

4. DESIGN PRINCIPLES

The nth order Chebyshev active filter can be constructed using cascade connection of stages. Each stage represents two conjugate poles of the Chebyshev active filter. While the real pole for odd orders , can be represented by single first order active filter stage.

Equation (8) gives the calculations of representing two conjugate poles by a second order active LPF of Figure (1). For odd orders , circuit of Figure (2) represents the single real pole.

For example if the order of filter is 7th order , then the designed stages can be shown in Figure (3):

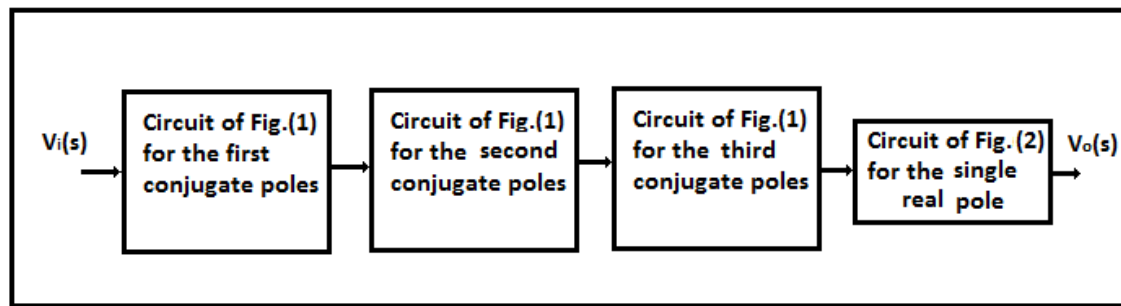


Figure (3) Stages of 7th order Chebyshev active LPF

The calculation of the transfer function of each stage depends on the calculated conjugate poles multiplication from Equation (8).

5. PROTOTYPE FILTER REALIZATION

The design of filter with calculated poles will give prototype filter with cut-off frequency 1rad./sec. , therefore realization steps must be given to create the real cut-off frequency and real filter components.

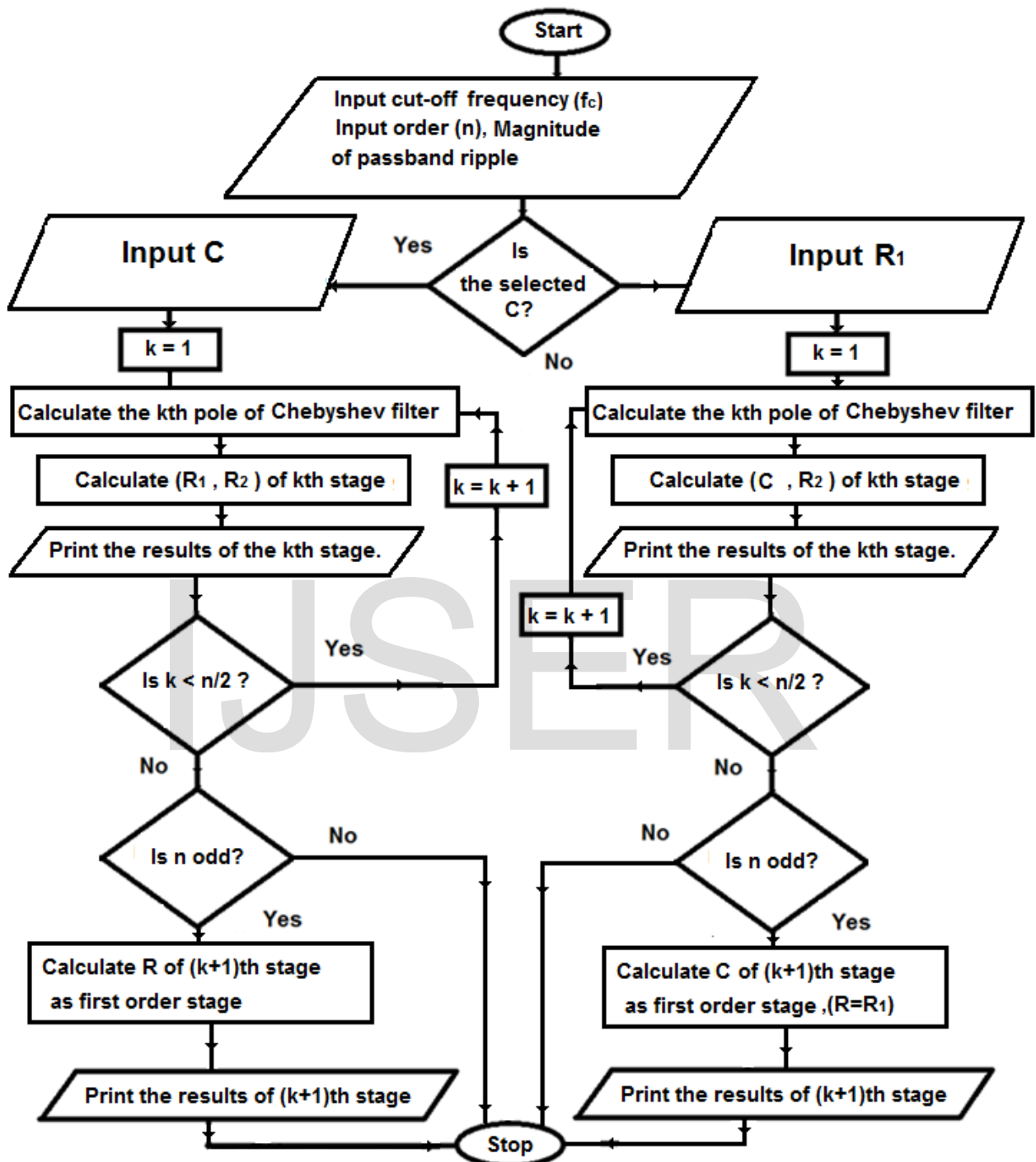
To get frequency transformation [1 rad./sec. to real cut-off frequency (f)] , each capacitance in the circuit must be divided by (2π.f) [3,10].

6. TRANSFORMATION OF FREQUENCY

In this paper we have only discussed the design of low pass filters (LPFs). There are many other types of filters that are used in signal processing. After the design of prototype filter, the other filters (High-pass, band-pass, and band-stop) can be created by a simple frequency transformation [9,11].

7. ALGORITHMS OF DESIGN

Flowchart (1) shows the algorithms of nth order Chebyshev active filter that give flexibility to choose the resistors or capacitors as well as other filter parameters. The algorithms are optimized to get highest possible accuracy and flexibility for choosing the order and components.



Flowchat(1): The algorithms of nth order Active Chebyshev Filter Design

8. DESIGN EXAMPLE RESULTS

Example results of the program (in C++ Language) which follows the algorithms of Flowchart (1) are as follows:

- For third order with freq.=66kHz, $A_p=0.5\text{dB}$ and with selected capacitance $C=1\text{nF}$
- The first stage: $R_1=3849.3\ \Omega$, $R_2=1322.3\ \Omega$
- The second stage : $R=7698.66\ \Omega$
- Fig.(4) shows above third order filter

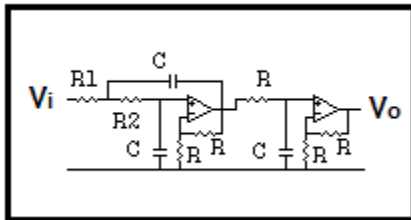


Fig.(4) Third order Chebyshev LPF

- For sixth order with freq.=66kHz, $A_p=0.5\text{dB}$ and with selected capacitance $C=1\text{nF}$
- The first stage: $R_1=15527.6\ \Omega$, $R_2=366.06\ \Omega$
- The second stage: $R_1=5683.49\ \Omega$, $R_2=1724.11\ \Omega$
- The third stage: $R_1=4160.6\ \Omega$, $R_2=8902.3\ \Omega$
- Fig.(5) shows above sixth order filter

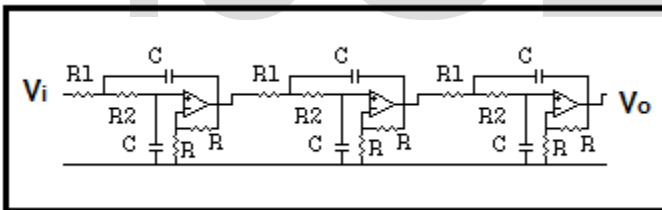


Fig.(5) Sixth order Chebyshev LPF

- For tenth order with freq.=66kHz, $A_p=0.5\text{dB}$ and with selected capacitance $C=1\text{nF}$
- The first stage: $R_1=43216.6\ \Omega$, $R_2=133.57\ \Omega$
- The second stage: $R_1=14891.44\ \Omega$, $R_2=472.92\ \Omega$
- The third stage: $R_1=9560.89\ \Omega$, $R_2=1143.66\ \Omega$
- The fourth stage: $R_1=7587.57\ \Omega$, $R_2=3221.28\ \Omega$
- The fifth stage: $R_1=6844.84\ \Omega$, $R_2=15095.34\ \Omega$
- Fig.(6) shows above tenth order filter

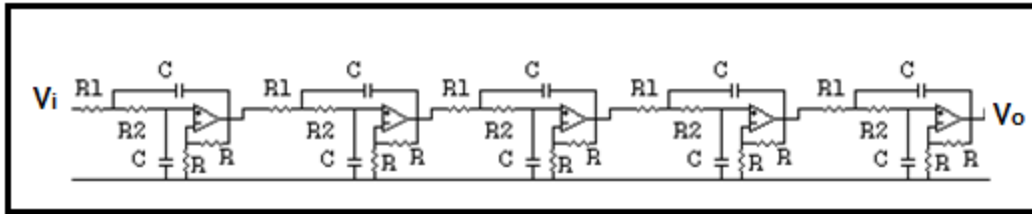


Fig.(6) Tenth order Chebyshev LPF

- For third order with freq.= 66kHz, $A_p=0.5\text{dB}$ and with selected resistance $R_1=6.6\text{k}\Omega$
- The first stage: $R_2=2267.19\Omega$, $C=0.58323\text{nF}$
- The second stage : $R=R_1$, $C=1.16646\text{nF}$
- Fig.(7) shows above third order filter

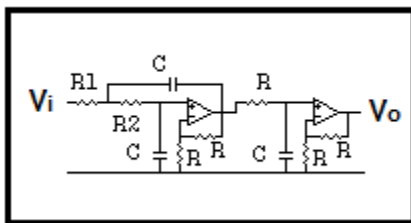


Fig.(7) Third order Chebyshev LPF

- For sixth order with freq.= 66kHz, $A_p=0.5\text{dB}$ and with selected resistance $R_1=6.6\text{k}\Omega$
- The first stage: $R_2=155.6\Omega$, $C=2.352667\text{nF}$
- The second stage: $R_2=2013.75\Omega$, $C=0.8611357\text{nF}$
- The third stage: $R_2=14121.8\Omega$, $C=0.630395\text{nF}$
- Fig.(8) shows above sixth order filter

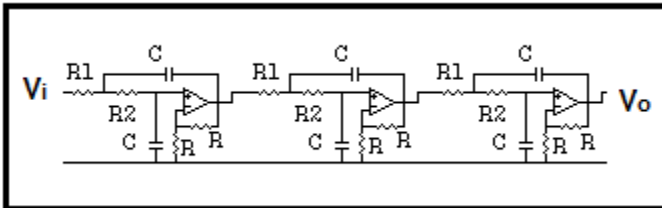


Fig.(8) Sixth order Chebyshev LPF

- For tenth order with freq.= 66kHz, $A_p=0.5\text{dB}$ and with selected resistance $R_1=6.6\text{k}\Omega$
- The first stage: $R_2=20.4\Omega$, $C=6.54798\text{nF}$
- The second stage: $R_2=209.6\Omega$, $C=2.25628\text{nF}$
- The third stage: $R_2=789.48\Omega$, $C=1.4486\text{nF}$
- The fourth stage: $R_2=2802\Omega$, $C=1.1496\text{nF}$
- The fifth stage: $R_2=14555.36\Omega$, $C=1.037\text{nF}$
- Fig.(9) shows above tenth order filter

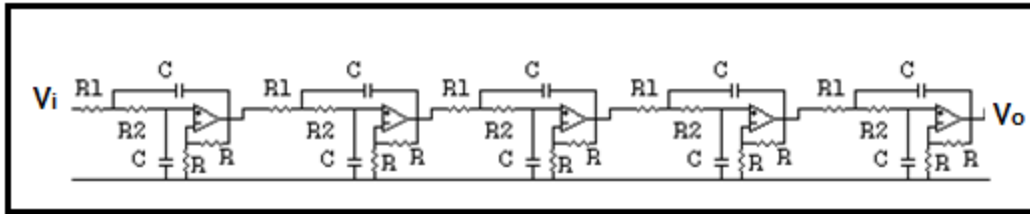


Fig.(9) Tenth order Chebyshev LPF

9. CONCLUSIONS

The program is simple to be used even without background in filtering principles. The performance of each designed filter was evaluated by "Electronic Workbench10", that resulted the design is exact and convenient for any practical application. The main outputs from this program are the components of each stage of the filter.

10. REFERENCES

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